ON THE NUMERICAL MODELING OF OPTICAL-SWITCHING IN NON-LINEAR PHASE-SHIFTED GRATING

A. SURYANTO

Department of Mathematics, Faculty of Mathematics and Natural Sciences, Brawijaya University
Jl. Veteran Malang 65145 Indonesia
suryanto@brawijaya.ac.id

Received 10 February 2009

We discuss the transmission properties of non-linear phase-shifted grating. Using a direct integration method, we show that phase-shift in linear sinusoidal grating introduces a very narrow resonance in the band gap with very high amplification factor. Using frequency in the vicinity of this resonance, we show that the threshold of switching can be reduced significantly. Based on the direct integration method, it is shown that Coupled-Mode Theory may produce inaccurate results for the case of both linear and non-linear grating structure with and without phase-shift.

Keywords: (Non-linear) phase-shifted sinusoidal grating; direct integration method; coupled-mode theory; optical switching.

1. Introduction

Periodically structured optical media have been in the focus of research activities for many years, due to their versatile technological applications in the fields of telecommunications and sensor systems (see e.g. Refs. 1–4). Transmission through linear periodic media is characterized by stop-band (or photonic band gap) centered at multiples of the corresponding Bragg frequency. The band gap is induced by the resonant coupling between the forward- and backward-propagating waves due to the Bragg resonance. In the presence of Kerr non-linearity where an increase on the field intensity changes the local refractive index, the entire photonic band is shifting, making the structure transparent at previously forbidden wavelengths. Even though this description is considered a simplified one, it basically explains the mechanism responsible for all optical-switching processes.\(^5\)

One of the important issues in realizing optical-switching is how to reduce intensity threshold. Previous studies have reported that the required switching intensities are extremely high, varying in the range 0.1–1GW/cm\(^2\) (see e.g. Refs. 6–8). Therefore, not only high-power sources are needed but thermal problems recognition connected with material absorption at such high intensities may occur. It
was shown theoretically using the Coupled-Mode Theory that phase-shifted grating leads to a narrow resonance centered in the middle of the Bragg stop-band. As the central resonance has to be shifted by approximately its width, one intuitively expects that switching may occur at much lower input intensity than that of uniform grating.\textsuperscript{5}

To study the behavior of light in linear grating theoretically, a lot of effort has been invested to develop (analytical or numerical) techniques such as Transfer-Matrix Method (TMM)\textsuperscript{9–11} and Coupled-Mode Theory (CMT).\textsuperscript{4,12–15} TMM is widely applied for step-index grating both linear (see e.g. Ref. 16) and non-linear grating (see, e.g. Ref. 17). Later, this method has been combined with TMM to analyze non-uniform non-linear sinusoidal grating, including phase-shifted grating.\textsuperscript{13} However, all these elaborated methods are based on the assumption that the medium can be divided into a number of homogeneous layers in which the wave equation solution is represented in the form of two counter-propagating plane waves. When these methods are implemented for non-linear grating, other assumptions are also applied, namely the slowly varying envelope approximation (SVEA) and the omission of spatial third harmonics generated in the structures. Alternatively, it is possible to use various numerical methods that solve the non-linear wave equations, e.g. the finite difference\textsuperscript{18} or the finite element\textsuperscript{19,20} methods but with continuation (iteration) method. The problem can also be solved by formulating it as initial value problem (IVP). Within this version, Baghdasaryan and Knyazyan\textsuperscript{21–23} represent the solution of the problem in the form of real amplitude and phase. In Ref. 24, Kwan and Lu propose a shooting method to solve the non-linear problem by forming an IVP directly for the electric field. Similar method has been applied for analyzing non-linear step-index grating.\textsuperscript{25–27} In this paper, we will implement such a method for analyzing non-linear sinusoidal grating with phase-shift. Since, apart from the numerical integration, the method is exact, the method will be used to assess the validity of Coupled-Mode Theory.

2. Formulation

2.1. Grating structure and direct integration method

Consider the grating structure whose index variation is given by

\[ n(z) = n_L(z) + \chi^{(3)}|E|^2, \]  

where the linear refractive index is \( n_L(z) = n_0 + \delta \sin(2\pi z / \Lambda) + \Omega(z) \) with \( \Lambda = 1/2n_0 \) and \( \chi^{(3)} \) is the third-order non-linearity. The grating phase \( \Omega(z) \) is assumed to be constant along the structure, except for discrete changes introduced by localized phase shifts \( \Delta \Omega \). For non-linear structures, a constant effective Kerr index \( \chi^{(3)} \) in the structures is considered.

It is well-known that the normally incident monochromatic electric field amplitude in one-dimensional Kerr medium satisfies the following non-linear Helmholtz
Numerical Modeling of Optical-Switching

**equation (NLH):**

\[ \frac{d^2 E}{dz^2} + k^2 n^2(z) E = 0, \]

(2)

where \( z \) is the propagation distance, \( k = \omega/c \) is the vacuum wave number, \( \omega \) is the angular frequency, \( c \) is the speed of light in free space and \( n \) is the refractive index.

With a transformation \( dE(z)/dz = Y(z) \), the NLH (2) can be written as a system of ordinary differential equations,

\[ \frac{dE(z)}{dz} = Y(z), \quad \frac{dV(z)}{dz} = -k^2 n^2(z) E(z). \]

(3)

To solve Eq. (3), boundary or initial conditions still have to be prescribed. For this, we assume that the background material of the grating is linear with refractive index \( n_0 \) without loss neither gain. Hence, the solution outside the grating can be written as superposition of forward and backward plane waves. If light with frequency \( \omega \) and amplitude \( A_{\text{inc}} \) is illuminated from the left, then it will generally be partly reflected and partly transmitted by the structure. By assuming that the grating structure is placed between \( z = 0 \) and \( z = z_{\text{max}} \), the electric field outside the structure can be expressed as follows:

\[ E(z) = \begin{cases} A_{\text{inc}} e^{-iknz} + A_{\text{refl}} e^{iknz}, & z \leq 0, \\ A_{\text{trans}} e^{-ikn(z-z_{\text{max}})}, & z \geq z_{\text{max}}, \end{cases} \]

(4)

where \( A_{\text{refl}} \) and \( A_{\text{trans}} \) are respectively the amplitude of the reflected and transmitted waves. Without loss of generality, we can assume that the transmitted wave amplitude \( A_{\text{trans}} \) is a real constant. From (4), we can then derive initial values for system (3), namely,

\[ E(z_{\text{max}}) = A_{\text{trans}}, \quad Y(z_{\text{max}}) = -ikn_0 A_{\text{trans}}. \]

(5)

Equation (3) together with initial values (5) forms a complete system and can be solved by any standard method for IVP. Here we employ the standard fourth-order Runge–Kutta method. The numerical integration is performed by backward calculation, i.e. from the non-illuminated side into the front border (\( z = 0 \)). By implementing Eq. (4) and the interface condition, the amplitude of incident and reflected waves can respectively be determined by the following formulas:

\[ A_{\text{inc}} = \frac{1}{2} \left( E(0) + \frac{i}{kn_0} Y(0) \right), \]

\[ A_{\text{refl}} = \frac{1}{2} \left( E(0) - \frac{i}{kn_0} Y(0) \right), \]

(6)
Finally, the reflectivity \( (R) \) and transmittivity \( (T) \) are calculated respectively using
\[
R = \left| \frac{A_{\text{refl}}}{A_{\text{inc}}} \right|^2 \quad \text{and} \quad T = \left| \frac{A_{\text{trans}}}{A_{\text{inc}}} \right|^2.
\] (7)

Since the IVP (3) and (5) are directly integrated, from now on, we call the method the Direct Integration Method (DIM).

### 2.2. Non-linear coupled-mode equations

To derive coupled-mode equations, first we assume that the modulation amplitude and the non-linearity is very small such that the square of refractive index (1) can be approximated by
\[
n(z)^2 = n_0^2 + 2n_0 \delta \sin \left( \frac{2\pi}{\Lambda} z + \Omega(z) \right) + 2n_0 \chi^{(3)} |E|^2.
\] (8)

Then the field within the structure is written in the standard form that separates the forward and the backward traveling waves as
\[
E(z) = A(z) \exp(-ik_0z) + B(z) \exp(ik_0z),
\] (9)
where \( k_0 = kn_0 \). Using the standard assumptions such as a slowly varying envelope approximation (SVEA) and the omission of spatial third harmonics generated in the structures, we obtain a set of non-linear coupled-mode equations
\[
\frac{\partial A}{\partial z} = \frac{1}{2} k\delta B \exp(\pm iq(2k_0 - K)z - i\Omega(z)) - i2n_0 \chi^{(3)} |A|^2 |B|^2 A,
\]
\[
\frac{\partial B}{\partial z} = \frac{1}{2} k\delta A \exp(\mp iq(2k_0 - K)z + i\Omega(z)) + i2n_0 \chi^{(3)} (|A|^2 + |B|^2) B,
\] (10)

where \( K = 2\pi/\Lambda \). The analytical solution of couple-mode equations for linear uniform grating is available in the literature (Yeh) and is given by
\[
A(z) = \frac{s \cosh[s(z_{\text{max}} - z)] + \frac{1}{2} \Delta k \sinh[s(z_{\text{max}} - z)]}{s \cosh[s(z_{\text{max}})] + \frac{1}{2} \Delta k \sinh[s(z_{\text{max}})]} A_0 \exp \left( \pm i\Delta k z \right),
\]
\[
B(z) = \frac{s \sinh[s(z_{\text{max}} - z)]}{s \cosh[s(z_{\text{max}})] + \frac{1}{2} \Delta k \sinh[s(z_{\text{max}})]} A_0 \exp \left( \mp \frac{i}{2} \Delta k z \right),
\] (11)

where \( \Delta k = 2k_0 - K, \kappa = (1/2)k\delta, s = \kappa^2 - (\Delta k/2)^2 \) and \( A_0 = A(0) \). For non-uniform non-linear grating, however, the exact solutions are generally difficult to be determined. In this paper, Eq. (10) is solved numerically using the same method as for DIM, i.e., using the fourth-order Runge-Kutta method. To solve Eq. (10)
numerically, boundary conditions are needed. Such boundary conditions can be derived using the same procedure as in the DIM and these are given by

\[
A(z_{\text{max}}) = A_{\text{trans}}, \\
B(z_{\text{max}}) = 0.
\]  

(12)

The reflectivity \(R\) and transmittivity \(T\) of the grating are calculated respectively using

\[
R = \left| \frac{B(0)}{A(0)} \right|^2 \quad \text{and} \quad T = \left| \frac{A(z_{\text{max}})}{A(0)} \right|^2.
\]  

(13)

3. Numerical Results and Discussion

3.1. Transmission characteristics of linear grating

The basic sinusoidal grating considered in this paper is a uniform grating with \(n_0 = 2\), \(\delta = 0.4\), \(\Lambda = 1/2n_0\), \(\Omega(z) = 0\) and \(\chi^{(3)} = 0\). The grid size used in the following simulations is \(\Delta z = \Lambda/100\). To see the accuracy of the fourth-order Runge–Kutta method, in Fig. 1, we compare the transmittivity of uniform grating with number of period \(N = 10\) obtained from numerical calculation of the CMT with that from exact calculation. It is shown that our numerical calculation agrees very well with the exact analytical solution. Furthermore, as shown in Fig. 1(b), the maximum absolute error of the numerical calculation is very small, i.e., in the order of \(10^{-11}\).

In Fig. 2, we compare the transmittivity of uniform grating obtained from DIM with that from CMT for two different number of periods. It is shown that CMT generally shifts the transmittivity to the right but with narrower side-lobes for frequency intervals at the left of the band gap and deeper side-lobe for frequency intervals at the right of the band gap. Other numerical simulations show that these differences disappear with the decreasing value of the grating amplitude modulation \(\delta\).

Figure 3 shows the field distribution inside the uniform grating with \(N = 20\) at the right-resonance band edge. It is shown that the incident field is amplified where the amplification factor is approximately 4.5. Figure 3(b) shows that CMT produces similar field distribution but with a slightly larger amplification factor.

We next consider linear grating structure with phase-shift. The phase-shift is introduced in the structure as follows:

\[
\Omega(z) = \begin{cases} 
0, & z < z_{\text{max}}/2, \\
\Delta\Omega, & z \geq z_{\text{max}}/2.
\end{cases}
\]  

(14)

Figure 4 compares the transmittivity of sinusoidal grating with \(N = 20\) and three different values of phase-shift. The most noteworthy feature is the appearance of the narrow transmission peak inside the band gap of uniform structure. It is seen that the narrowest resonance is achieved when phase-shift \(\Delta\Omega = \pi\). For this resonance, the grating structure is transparent, and the corresponding distribution of
the electric field is presented in Fig. 5. The phase-shift inserted in the middle of the structure acts as a divider of the grating into separated high-reflective Bragg grating mirrors. Therefore, the structure produces typical properties of the Fabry–Perrot interferometer, which is evident from the electric field distribution in the structure. Here, the amplification factor of the incident wave is approximately 22, which is much larger compared to that of uniform grating of the same size. Outside the structure, the traveling waves with constant amplitudes are observed, indicating the case of total transmission. We observe from our other simulations that the resonance peak is narrower and the amplification factor of the electric field is greater when the number of period grating is larger.

In Fig. 4, we also compare the transmittivity obtained from DIM with that from CMT. Qualitatively, CMT can capture the behavior of phase-shifted grating. Indeed, similar to the case of uniform grating, CMT shifts the transmittivity to the right. Surprisingly, the resonance frequency inside the band gap is not shifted to the right but to the left. The field distribution inside the structure at resonance frequency obtained from DIM and CMT also shows similar behavior. To see this behavior, in Fig. 5, we compare the electric field at resonance for $\Delta \Omega = \pi$ calculated by DIM and CMT. Both DIM and CMT produce the same profile but CMT calculation gives a slightly higher amplification factor.
Fig. 2. Transmittivity of uniform grating with (a) $N = 10$ and (b) $N = 20$ calculated by DIM and CMT.
Fig. 3. Field distribution inside uniform grating ($N = 20$) at resonance frequency (right-band edge) calculated by (a) DIM and (b) CMT.

Fig. 4. Transmittivity of linear phase-shifted grating for different values of phase-shifted $\Delta \Omega$ calculated by DIM and CMT.
3.2. Input-output characteristics of non-linear grating

If Kerr non-linearity is introduced in the structure, it causes a change of local refractive index and obviously it will also alter the transmission spectrum. Specifically, when the resonance is sharp enough, a bistability phenomenon can be obtained. A basic issue of the optical bistability is to realize it with a threshold as low as possible. A threshold requires a large non-linear effect. The non-linear effect can be increased by choosing a structure that has a high field amplitude enhancement factor and selecting the input frequency of light in the vicinity of a resonance frequency. In the following discussion, we study effects of phase-shift on the characteristics of the optical-switching. We assume a uniform Kerr non-linearity with $\chi^{(3)} = 2 \times 10^{-12} \text{m}^2\text{V}^{-2}$ present in the grating structure.

Using DIM, it is possible to obtain complete characteristics of sinusoidal grating. Figure 6 shows these characteristics for uniform grating with $N = 20$. It is evident from Fig. 6 that a bistable behavior can be expected by detuning the frequency slightly at the left of the right band edge. In Fig. 6, we also compare the results of DIM with that of CMT. It is found that CMT produces relatively high error compared to DIM. Indeed, the threshold for up-switching obtained by DIM is approximately 1266 kW/m$^2$ while CMT needs 1132 kW/m$^2$. Furthermore, the frequency used in these calculations is not the same because it should be tuned near the band edge resonance of linear grating. As the band edge is shifted to the right by CMT, the frequency used in CMT is larger than that used in DIM calculation. Figures 6(b) and 6(c) shows the comparison of the electric field inside the structure obtained by DIM and CMT at resonance (input intensity = output intensity). It is
observed that the amplification factor obtained by CMT is larger than that obtained by DIM. This intuitively explains that the threshold of up-switching calculated by CMT is smaller than that calculated by DIM.

As shown before, when phase-shift is introduced in uniform grating, it can produce a very narrow resonance inside the band gap with a relatively high amplification factor. Therefore, it can be used to enhance non-linear effects. To get the maximum amplification factor, we choose phase shift $\Delta \Omega = \pi$. To obtain fair comparison with uniform grating structure discussed previously, we also take $N = 20$. Similar to the previous case, the input frequency is chosen in the vicinity of resonance. Using transmittivity of the linear grating, we take input frequency $\omega = 1.016 \times 2\pi c/\lambda_0$ for DIM calculation and $\omega = 0.9988 \times 2\pi c/\lambda_0$ for CMT calculation. Figure 7 shows input–output characteristics of phase-shifted grating. As expected, the bistability threshold is much lower compared to a uniform grating of the same size. Here, the threshold is extremely low and is approximately $14.5 \text{kW/m}^2$. Figure 7 also shows that the calculation of CMT has good agreement with that of DIM. However, it should be noted that the frequencies of the input wave used in CMT and DIM are different. If for CMT calculation we choose input frequency as used by DIM, then
there is no bistable phenomena observed in the input–output curve except with very high input intensity threshold. (This bistability is not caused by resonance frequency in the band gap.) At resonance, the electric field distribution inside the structure calculated by CMT also has a good agreement with that by DIM.

4. Conclusion
We have studied the optical response of linear and non-linear sinusoidal grating. The transmission properties of a phase-shifted structure have been used to obtain extremely low switching intensities. In this paper, we solve the non-linear Helmholtz equation by representing it as a system of first-order differential equations. This system can be solved directly by standard Runge–Kutta. Since this formulation does not include any approximation except the numerical integration, the method is used to see the validity of the Coupled-Mode Theory which is widely used in the literature. We show that Coupled-Mode Theory may produce inaccurate results for the case of both linear and non-linear grating structure with and without phase-shift.
A. Suryanto

Acknowledgment

The authors would like to thank Direktorat Penelitian dan Pengabdian kepada Masyarakat, Direktorat Jenderal Pendidikan Tinggi Indonesia (No. 320/SP2H/PP/DP2M/III/2008), for financial support.

References